

# Il numero e il contare nel mondo infantile

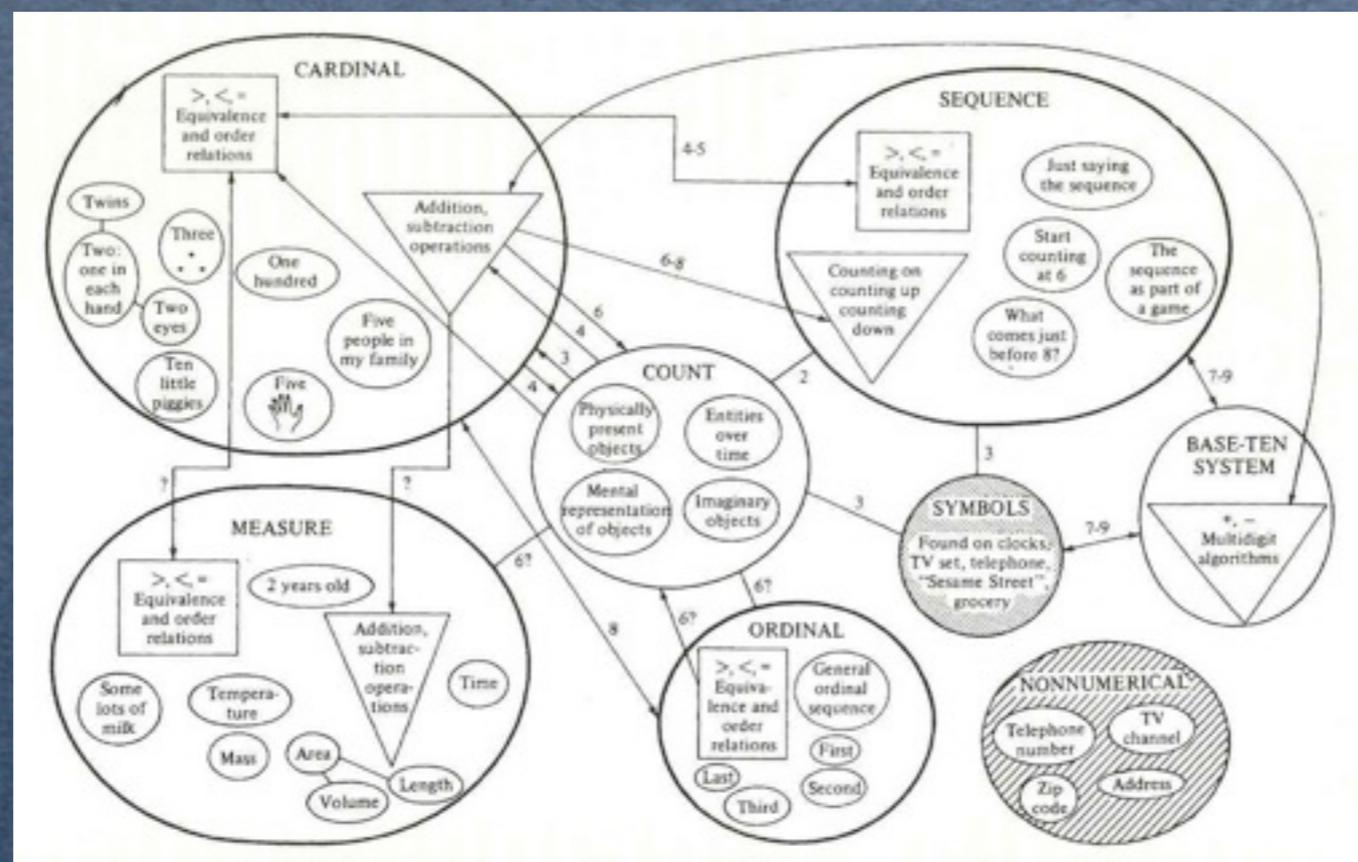
Karen Fuson  
Northwestern Universty  
Università Roma Tre  
16 ottobre 2012

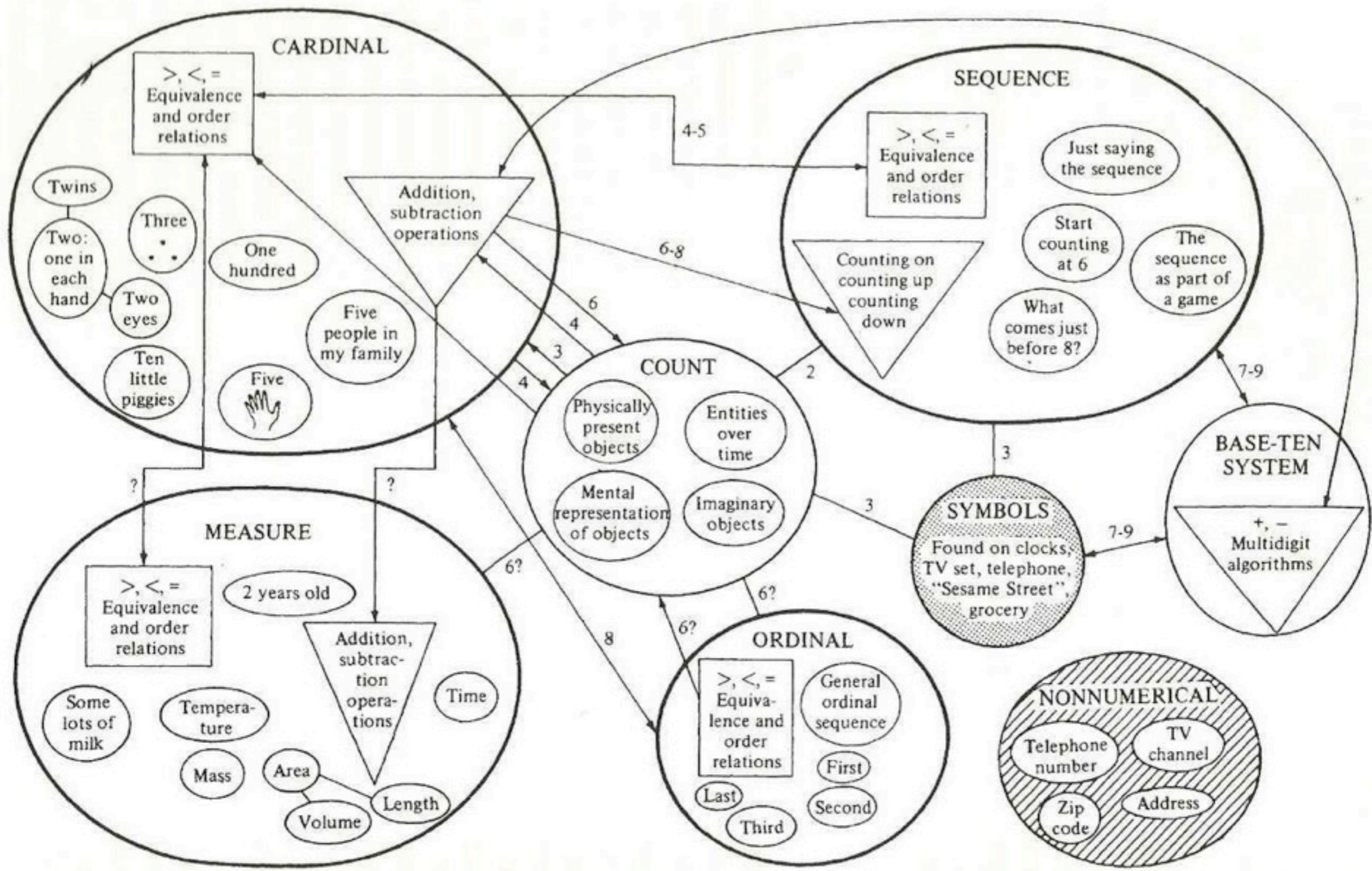
# Il numero nel mondo infantile

- 1 - I molti significati dei vocaboli numerali
- 2 - Prime idee numeriche
- 3 - I livelli nell'addizione e nella sottrazione
- 4 - Comprendere i numeri a più cifre e le addizioni e sottrazioni con numeri a più cifre

# 1- I molti significati dei vocaboli numerali

# Una mappa mentale approssimativa delle prime esperienze infantili con le parole numero





**CARDINAL**

**SEQUENCE**

**COUNT**

**MEASURE**

**ORDINAL**

**SYMBOLS**

**BASE-TEN SYSTEM**

**NONNUMERICAL**

>, <, =  
Equivalence  
and order  
relations

+, -  
Multidigit  
algorithms

Addition,  
subtraction  
operations

Counting on  
counting up  
counting down

Addition,  
subtraction  
operations

Telephone  
number  
TV  
channel  
Zip  
code  
Address

Twins

Two:  
one in  
each  
hand

Two  
eyes

Ten  
little  
piggies

Three  
•  
•

One  
hundred

Five

Five  
people in  
my family

2 years old

Some  
lots of  
milk

Tempera-  
ture

Mass

Area

Volume

Length

Time

Physically  
present  
objects

Entities  
over  
time

Mental  
representation  
of objects

Imaginary  
objects

General  
ordinal  
sequence

First

Second

Last

Third

4-5

6-8

6

4

3

4

2

3

3

6?

6?

8

6?

7-9

7-9

?

?

# Usi dei vocaboli numerali

- uso cardinale
- uso ordinale
- uso nella misura
- uso sequenziale
- contare
- uso per nominare i simboli (cifre)
- etichette o codici

USI DELLE PAROLE NUMERALI	SITUAZIONE IN RELAZIONE A OGGETTI	ESISTE UN ORDINAMENTO?	ESISTONO UNITA'?	REFERENTE
Cardinale	Entità discrete	No	Unità percettiva	L'intera famiglia di entità
Ordinale	Entità discrete	Si	Unità percettiva	Un'entità in una famiglia
Misura	Una quantità continua	No	Unità di quella quantità continua	La quantità continua
Sequenza	Nessuna	Si	Cambiamenti nel corso dello sviluppo	Nessun riferimento
Conteggio	Entità discrete	No	Unità percettiva	Ogni entità
Simboli (cifra)	Un simbolo	No	No	Quel simbolo: potrebbe anche riferirsi ad uno degli altri usi
Etichette o serie non numeriche	Una singola entità	No	No	Quell'entità

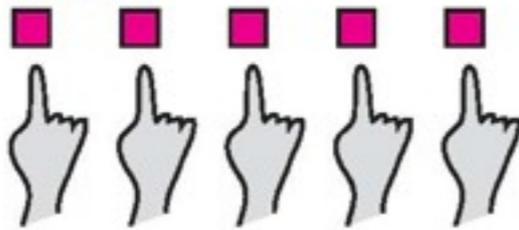
USI DELLE PAROLE NUMERALI	LE PAROLE DESCRIVONO ....	RELAZIONE DI EQUIVALENZA	RELAZIONI D'ORDINE	OPERAZIONI
Cardinale	La numerosità del gruppo di entità discrete	Lo stesso numero che	Più di/meno di	+ , - di numeri interi
Ordinale	La posizione relativa dell'entità	La stessa posizione relativa di	Prima di/dietro, nell'ordine	+ , - dei numeri ordinali
Misura	La numerosità delle unità che coprono (riempiono) la quantità	La stessa quantità	Più di/ meno di	+ , - i numeri interi, decimali, frazioni
Sequenza	Niente	La stessa parola	Dopo/prima Appena dopo/ appena prima	Parole della sequenza usate per + , - dei numeri interi
Conteggio	Niente	Nessuna	Nessuna	Nessuna
Simboli (cifra)	Quel simbolo; qualsiasi sia l'utilizzo di quel simbolo	Solo se interpretato come cardinale o misura	Solo se interpretato come cardinale o misura	Solo se interpretato come cardinale o misura
Etichette o serie non numeriche	Vari attributi di un'entità	No	No	No

# 2 - Prime idee numeriche

## The last number word tells how many

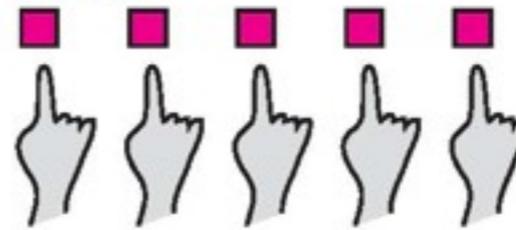
Teacher: "How many blocks are there?"

Child 1:



"1" "2" "3" "4" "5"

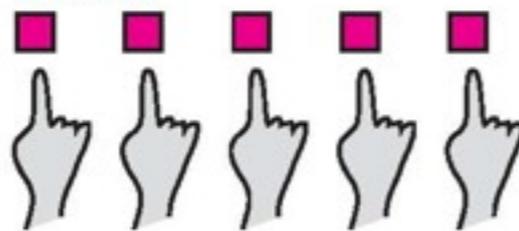
Child 2:



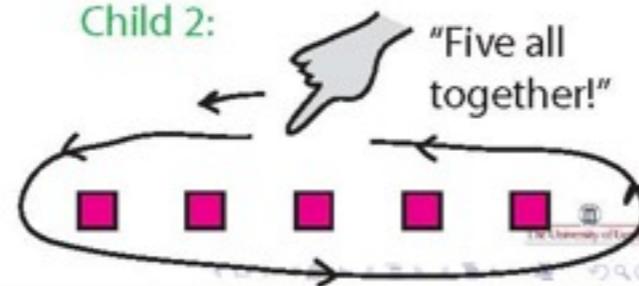
"1" "2" "3" "4" "5"

Teacher: "So how many blocks are there?"

Child 1:



Child 2:

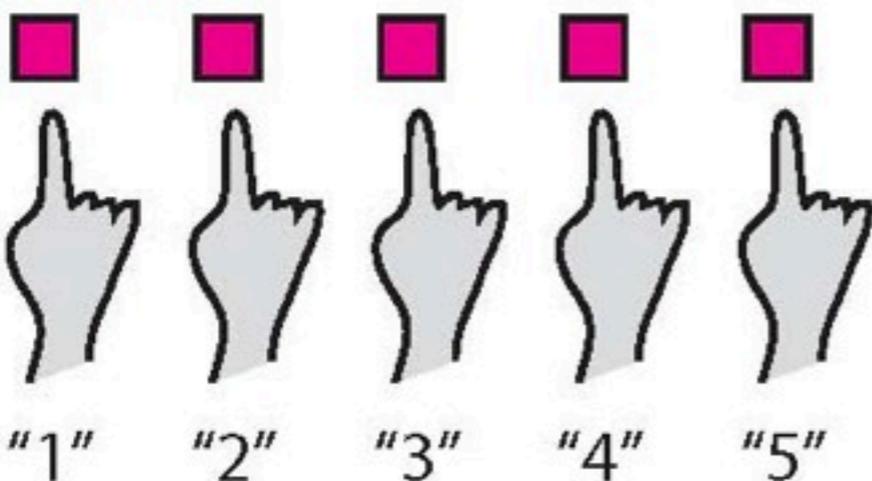


# La transizione dal contare al cardinale

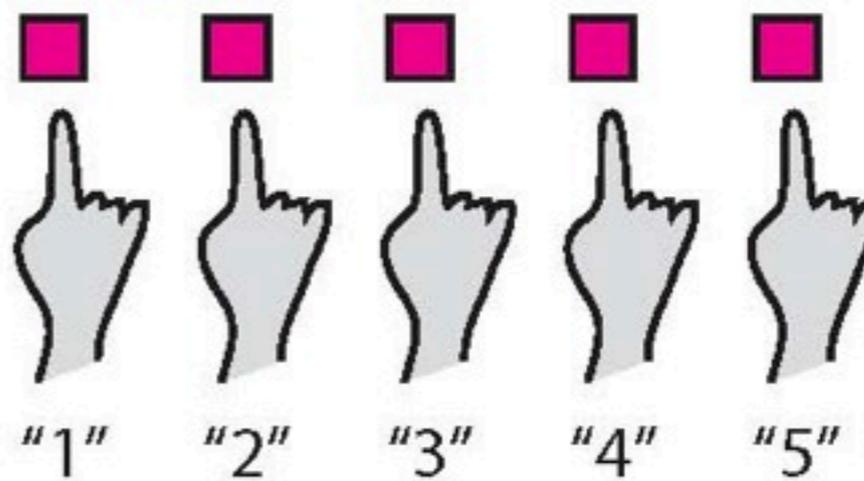
## The last number word tells how many

Teacher: "How many blocks are there?"

Child 1:

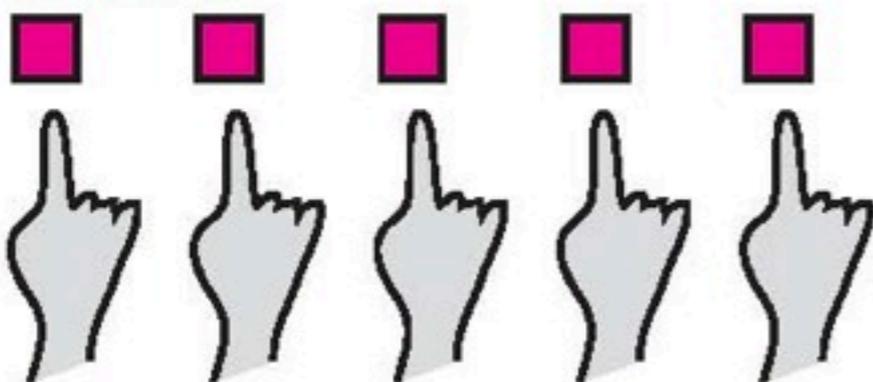


Child 2:

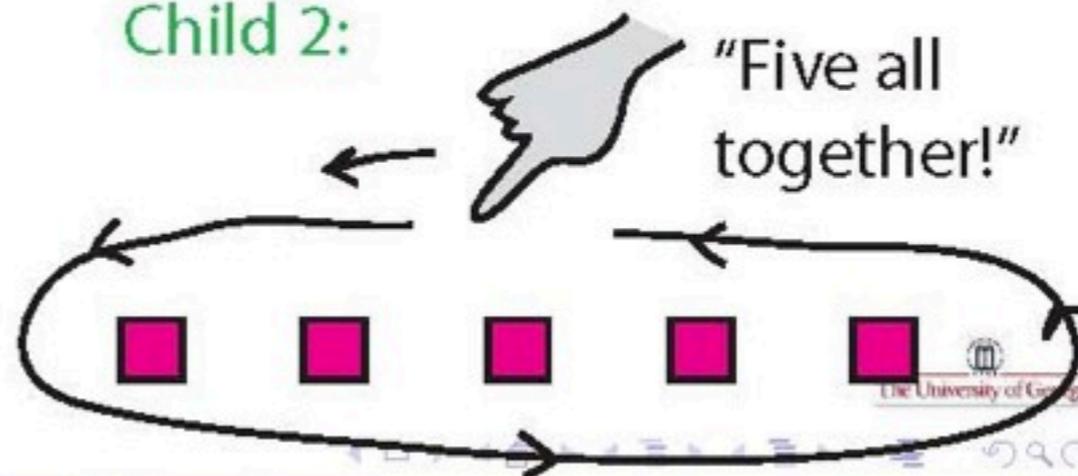


Teacher: "So how many blocks are there?"

Child 1:



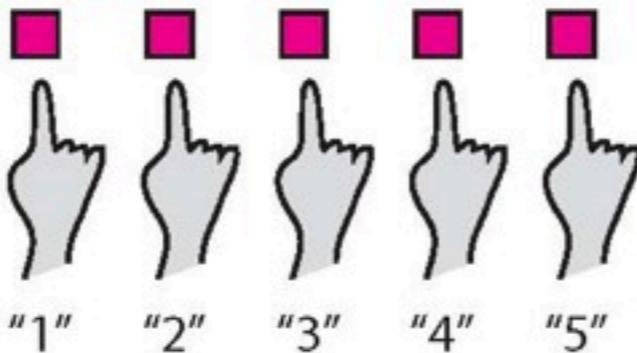
Child 2:



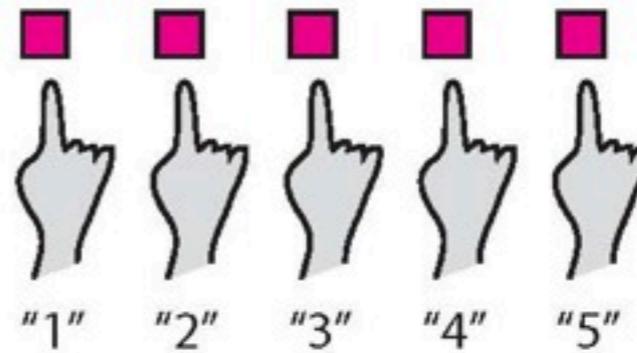
## The last number word tells how many

Teacher: "How many blocks are there?"

Child 1:

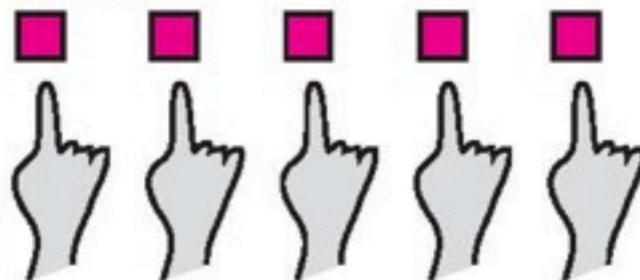


Child 2:

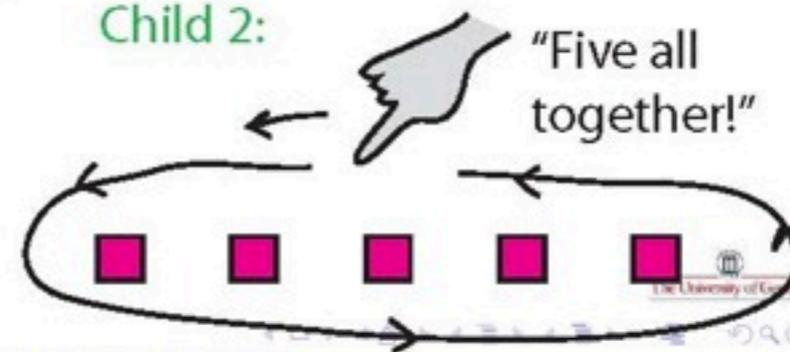


Teacher: "So how many blocks are there?"

Child 1:



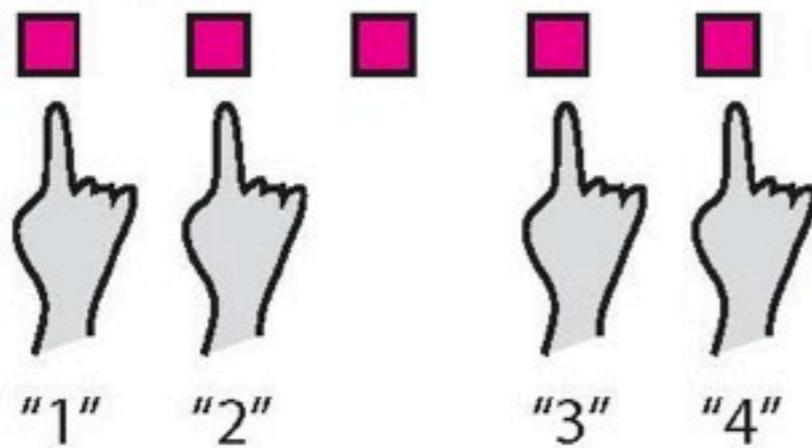
Child 2:



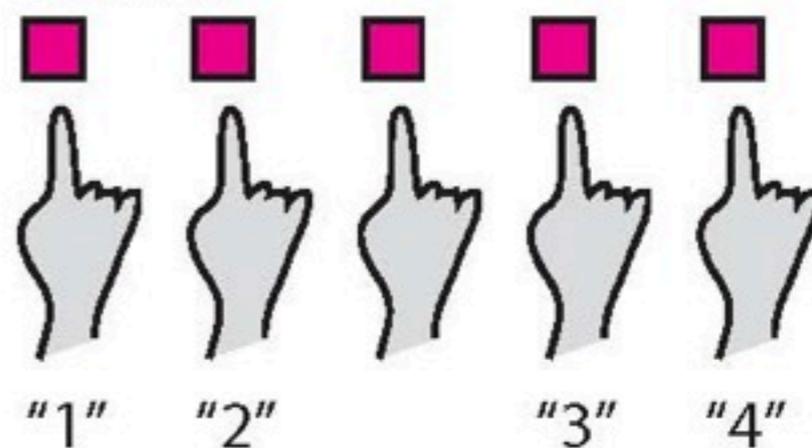
# Gli errori nel contare

# One-to-one correspondence errors

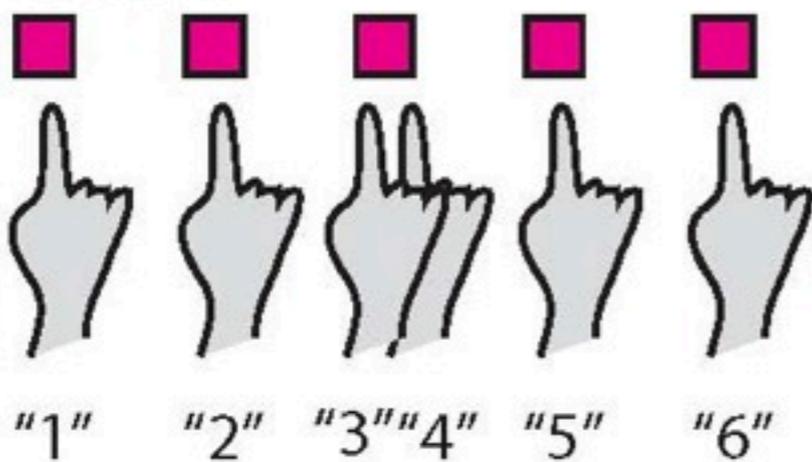
Child 1:



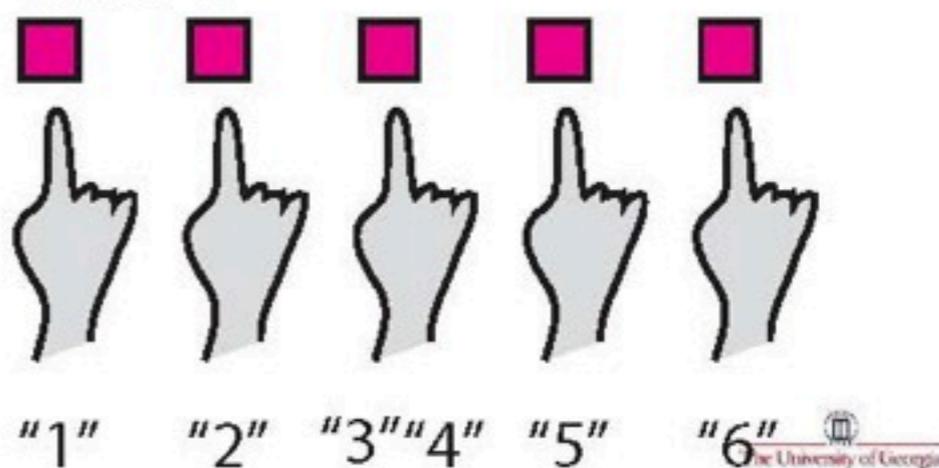
Child 2:



Child 3:



Child 4:



**Strategie più/meno  
e  
contare/appaiare**

*Sembra di più.*

O O O O O O O

O O O O O O O

*In questa riga sembra che vi siano di più.*

*Vi sono veramente di più? Conterò per verificare*

1 2 3 4 5 6 7

O O O O O O O

O O O O O O O

1 2 3 4 5 6 7

*Ho contato: ogni riga: ogni riga ha sette quindi in realtà hanno uguale.*

*Vi sono veramente di più? Collego (accoppio) per verificare.*

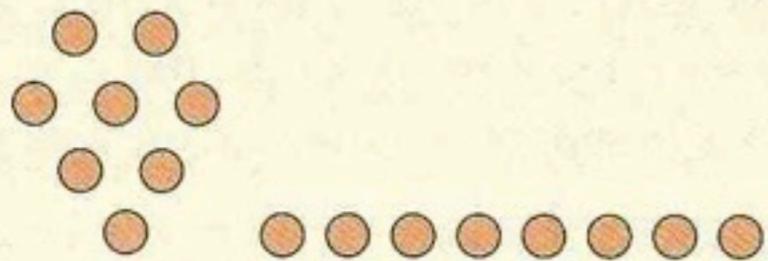


*Ho fatto combaciare le due righe.*

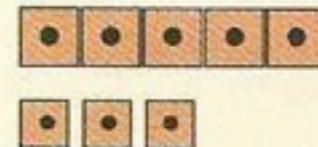
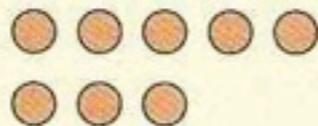
*Non rimane nulla. Quindi quindi in realtà hanno uguale*

Sembra di più oppure vi sono di più?

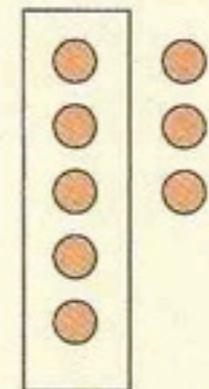
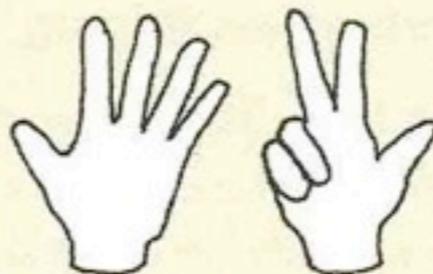
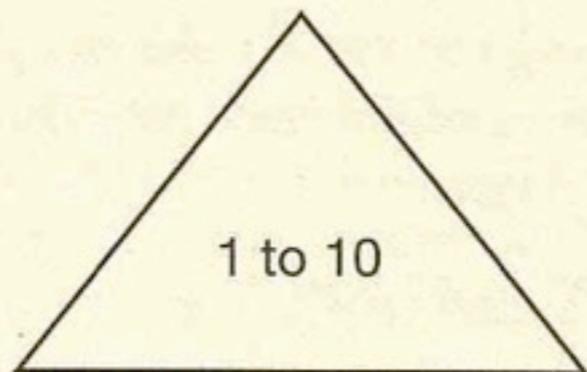
Count



See 5-groups



Quantities

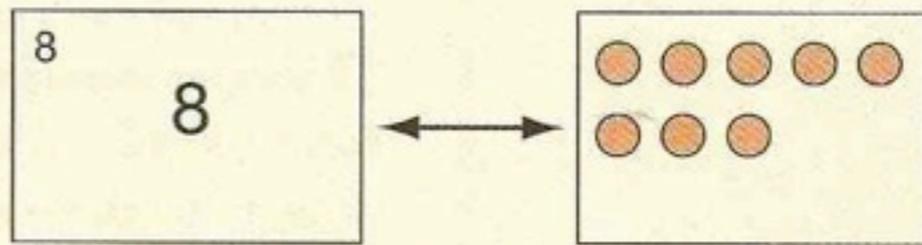


Count Words

Written Numerals

Eight

8



Da 1 a 10

Quantità  
Parole numerali  
Simboli scritti

**3 – I livelli  
nell'addizione e nella  
sottrazione**

# Metodi per l'addizione e per la sottrazione

Livello 1: Contare (tutto)

Livello 2: Contare da

Livello 3: Ricomporre, fare dieci

**CONTARE DA RENDE LA SOTTRAZIONE  
SEMPLICE QUANTO L'ADDIZIONE**

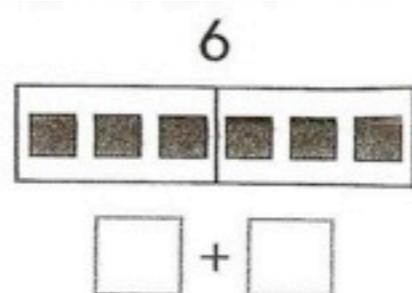
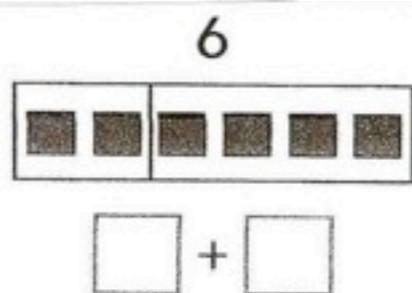
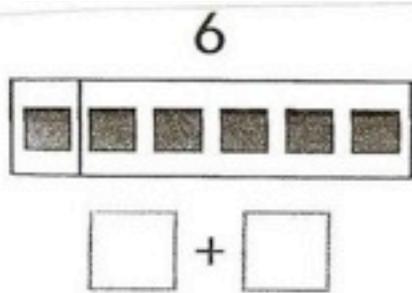
**Table 3 Levels of Children's Addition and Subtraction Methods**

	$8 + 6 = 14$	$14 - 8 = 6$																
Level 1: Count all	<p style="text-align: center;"><b>Count All</b></p> <p>a</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%;">1 2 3 4 5 6 7 8</td> <td style="width: 50%;">b</td> </tr> <tr> <td>○ ○ ○ ○ ○ ○ ○ ○</td> <td>1 2 3 4 5 6</td> </tr> <tr> <td>1 2 3 4 5 6 7 8</td> <td>○ ○ ○ ○ ○ ○</td> </tr> <tr> <td>c</td> <td>9 10 11 12 13 14</td> </tr> </table>	1 2 3 4 5 6 7 8	b	○ ○ ○ ○ ○ ○ ○ ○	1 2 3 4 5 6	1 2 3 4 5 6 7 8	○ ○ ○ ○ ○ ○	c	9 10 11 12 13 14	<p style="text-align: center;"><b>Take Away</b></p> <p>a</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%;">1 2 3 4 5 6 7 8 9 10</td> <td style="width: 50%;">11 12 13 14</td> </tr> <tr> <td><del>○ ○ ○ ○ ○ ○ ○ ○ ○ ○</del></td> <td>○ ○ ○ ○</td> </tr> <tr> <td>1 2 3 4 5 6 7 8 1 2 3 4 5 6</td> <td></td> </tr> <tr> <td>b</td> <td>c</td> </tr> </table>	1 2 3 4 5 6 7 8 9 10	11 12 13 14	<del>○ ○ ○ ○ ○ ○ ○ ○ ○ ○</del>	○ ○ ○ ○	1 2 3 4 5 6 7 8 1 2 3 4 5 6		b	c
1 2 3 4 5 6 7 8	b																	
○ ○ ○ ○ ○ ○ ○ ○	1 2 3 4 5 6																	
1 2 3 4 5 6 7 8	○ ○ ○ ○ ○ ○																	
c	9 10 11 12 13 14																	
1 2 3 4 5 6 7 8 9 10	11 12 13 14																	
<del>○ ○ ○ ○ ○ ○ ○ ○ ○ ○</del>	○ ○ ○ ○																	
1 2 3 4 5 6 7 8 1 2 3 4 5 6																		
b	c																	
Level 2: Count on	<p style="text-align: center;"><b>Count On</b></p>	<p style="text-align: center;">To solve <math>14 - 8</math> I count on <math>8 + ? = 14</math></p>																
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend	<p style="text-align: center;"><b>Recompose: Make a Ten</b></p>	<p style="text-align: center;"><math>14 - 8</math>: I make a ten for <math>8 + ? = 14</math></p>																
Make a ten (from 5's within each addend)		<p style="text-align: center;"><math>8 + 6 = 14</math></p>																
Doubles $\pm n$	<p style="text-align: center;"><math>6 + 8</math> <math>= 6 + 6 + 2</math> <math>= 12 + 2 = 14</math></p>																	

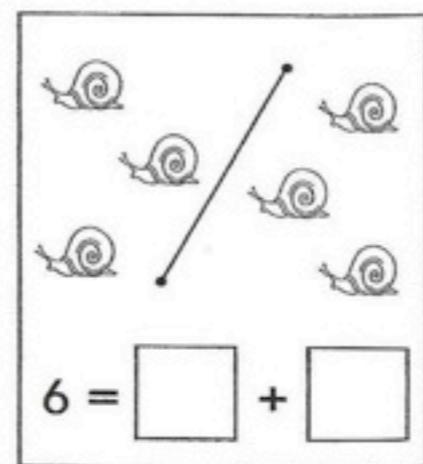
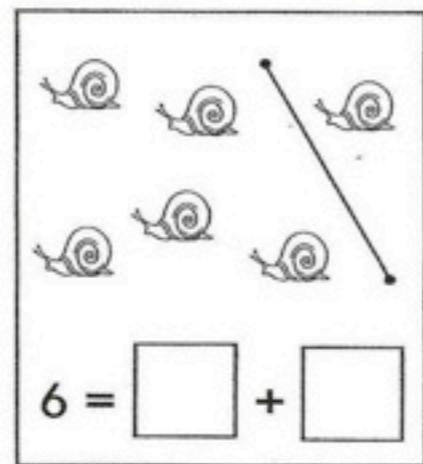
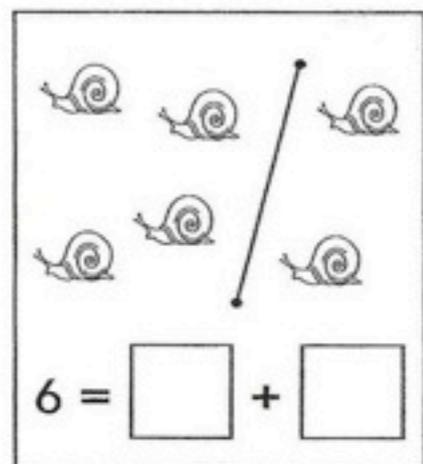
Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

# Cercare e scrivere i “duetti”

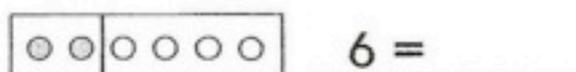
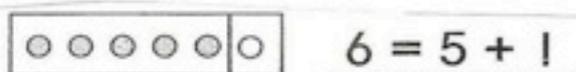
“Duetto” (partners) è una parola infantile per “addendi” in una decomposizione di un numero (un totale) usando due addendi



**B. Write the partners.** [Write the partner numbers in an equation.]



**C. Write the partner equation.** [Write a partner expression in an equation.]



D. Same as B or C but children draw a line in each picture and finish the equation by writing partners for the B version and writing an expression for the C version.

Una uguaglianza esprime bene la situazione (5/6 anni)

# Livelli nella sequenza del contare

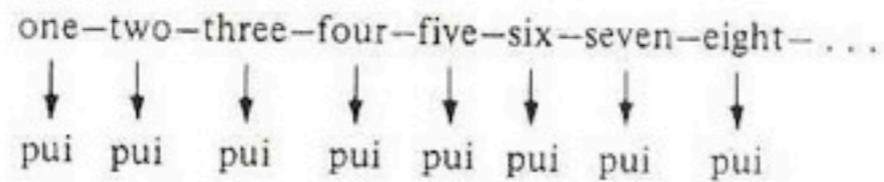
Level

Sequence

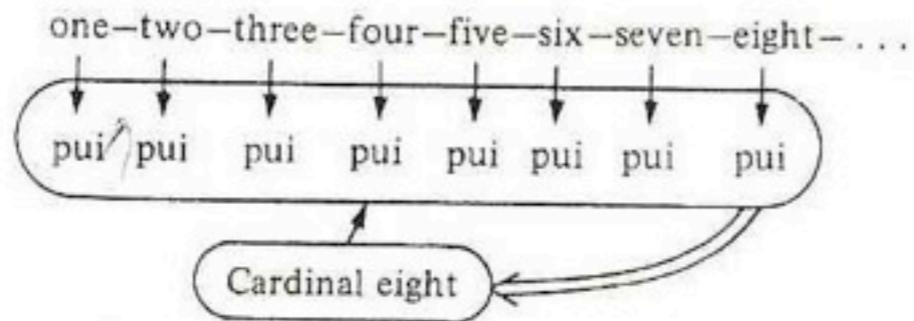
String

Onetwothreefourfivesixseveneight . . .

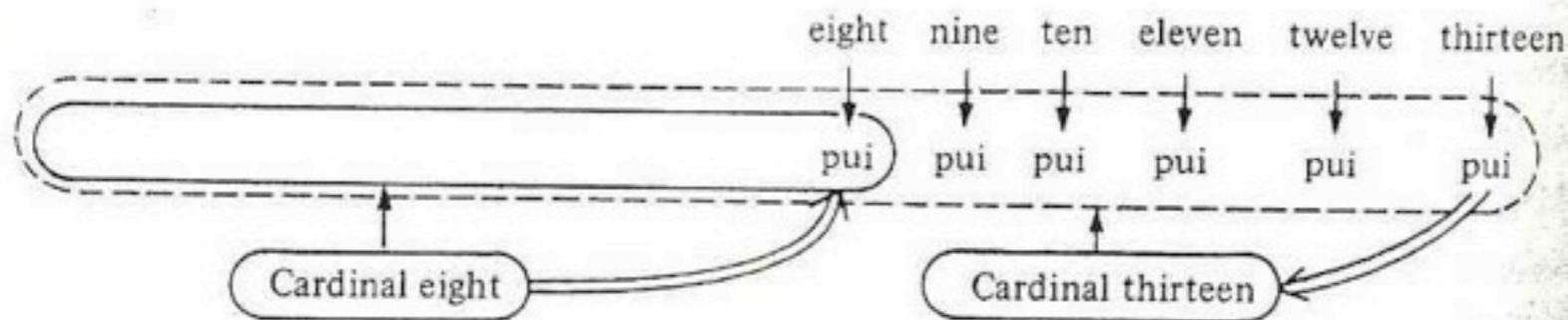
Unbreakable list



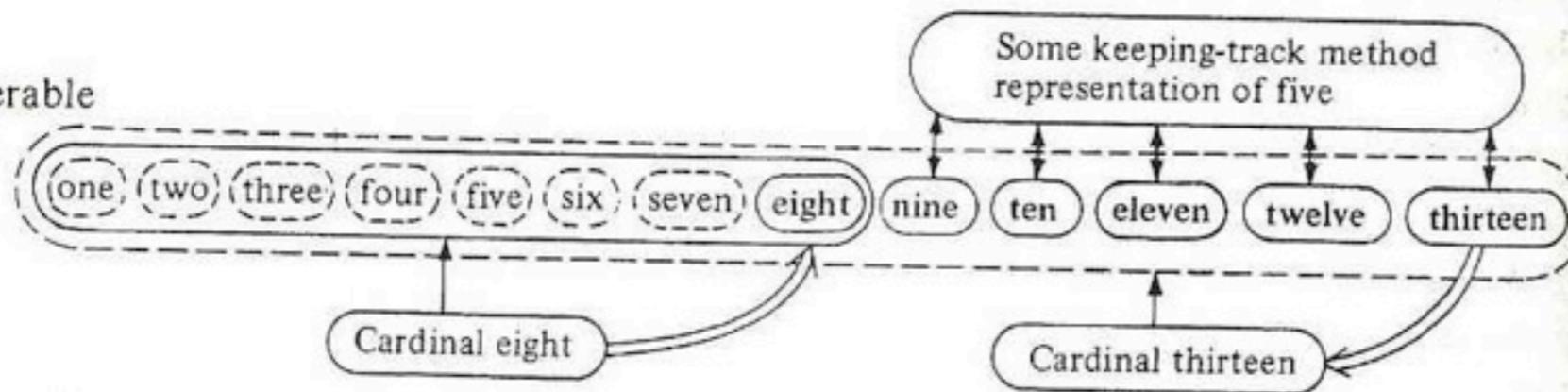
Unbreakable list



Breakable chain



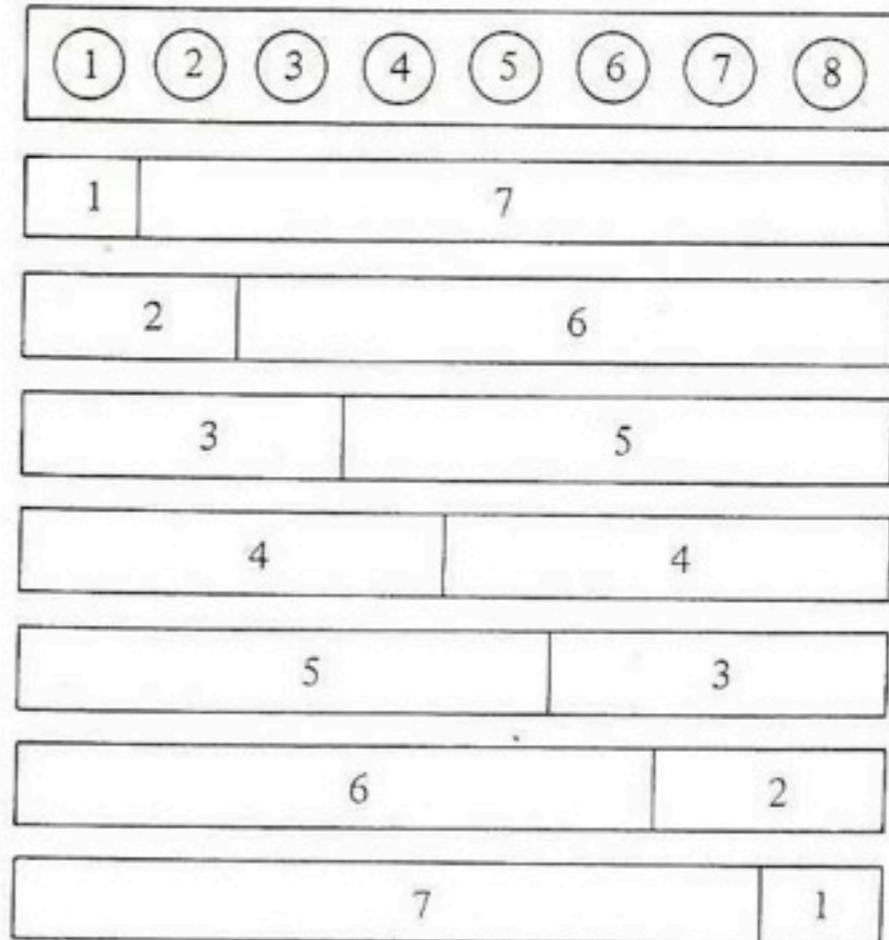
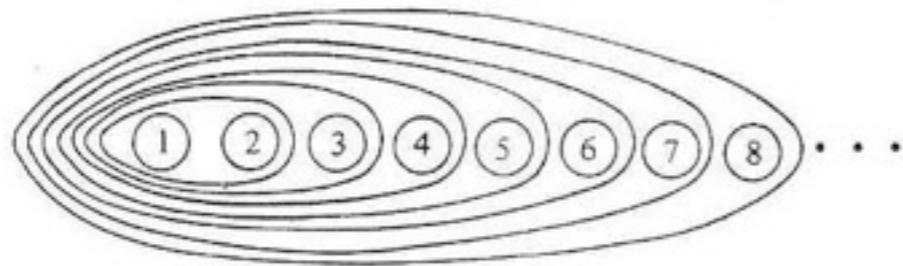
Numerable chain



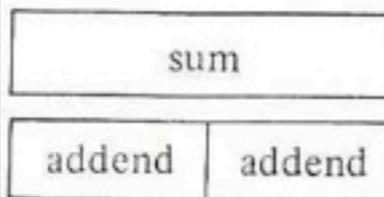
pui: item unitario percepito

La sequenza orale: contare e sommare

Bidirectional chain/  
truly  
numerical  
counting

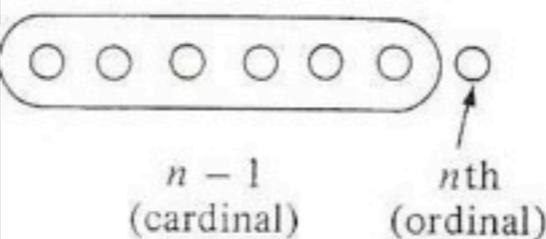


# Genuino contare numerico (catena bidirezionale)

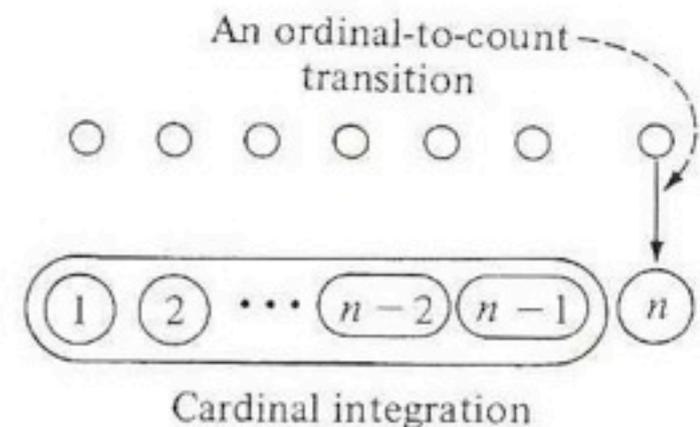


$\text{addend} + \text{addend} = \text{sum}$

$\text{sum} - \text{addend} = \text{addend}$



via the  
numerical  
sequence

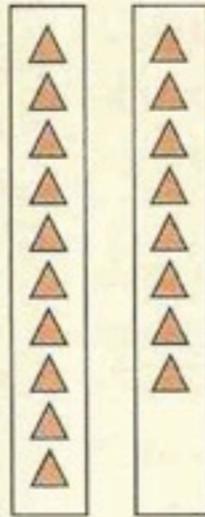


Cardinal integration

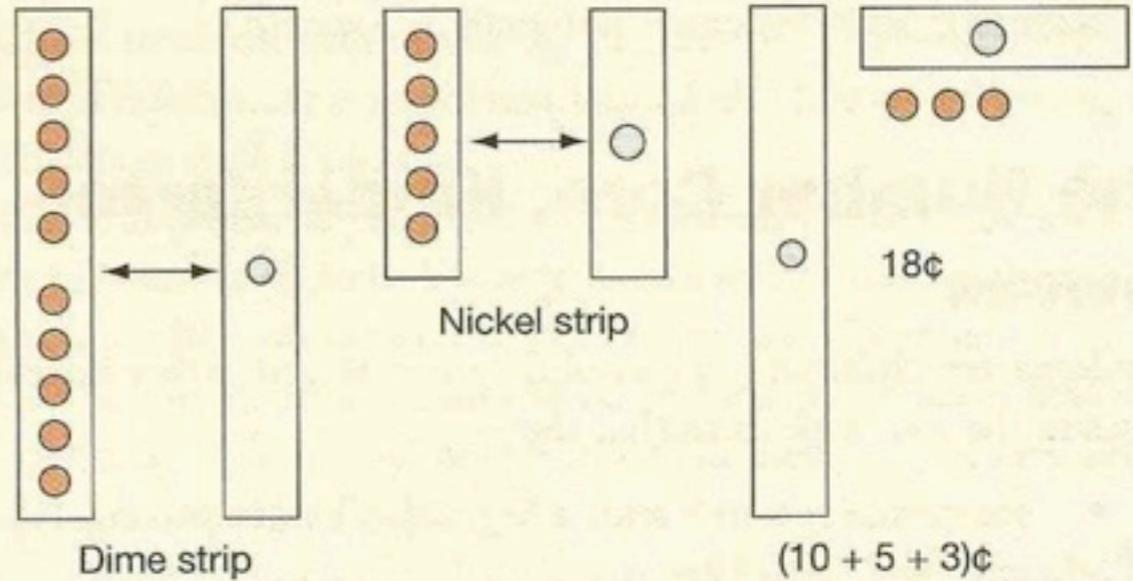
# Da 10 a 19

Quantità  
Parole numerali  
Simboli scritti

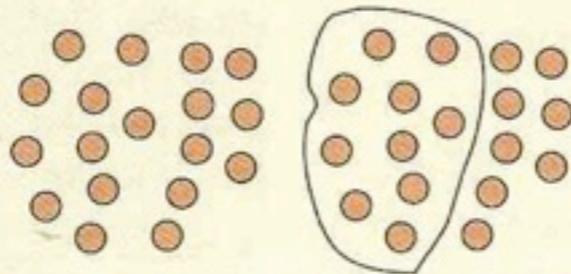
B. See ten and some ones



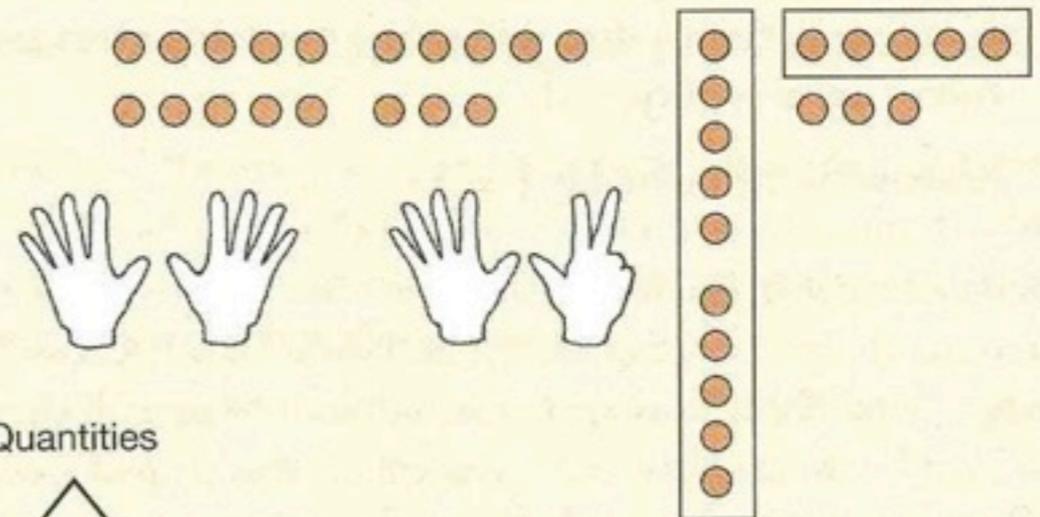
D. Coin values as a 5-group of pennies = a nickel and two 5-groups = a dime



A. Unitary



C. See 5-groups within ten and the ones



Quantities

10 to 19

Count Words

Written Numerals

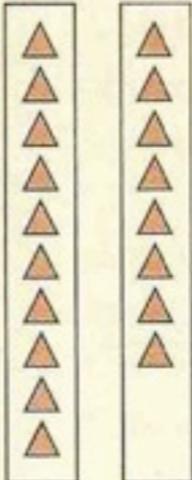
five, thirteen, fourteen, fifteen,  
teen, eighteen, nineteen  
en]

The ones are first in the words and  
second in the written numerals.

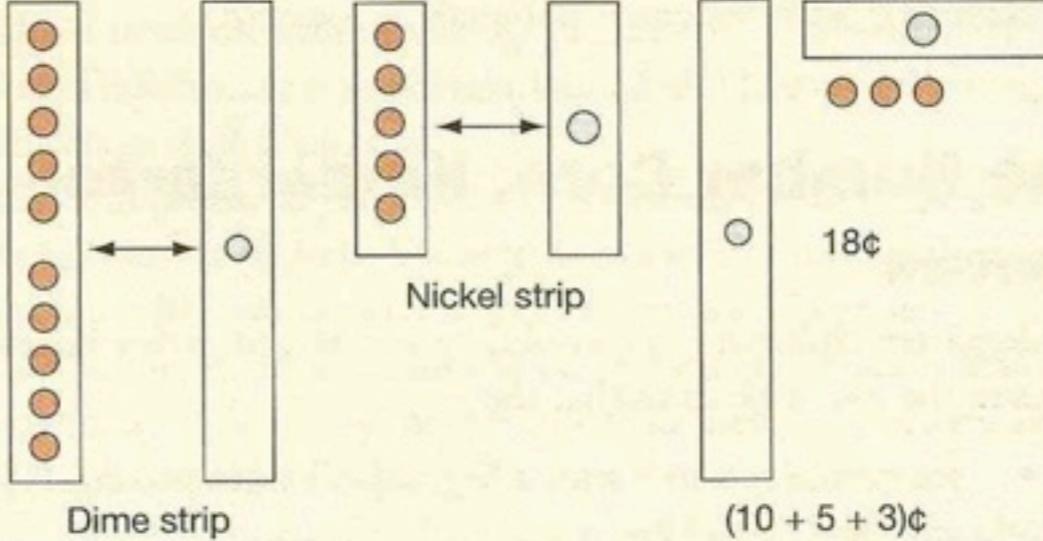
B,C,D  
Dieci e otto  
Una decina  
otto unità

A >  
Diciotto

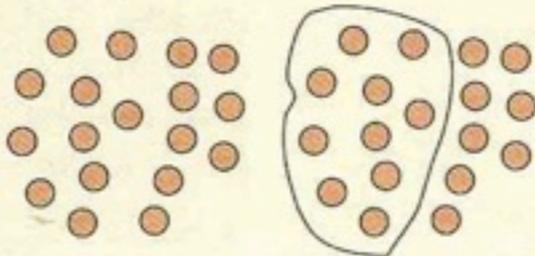
B. See ten and some ones



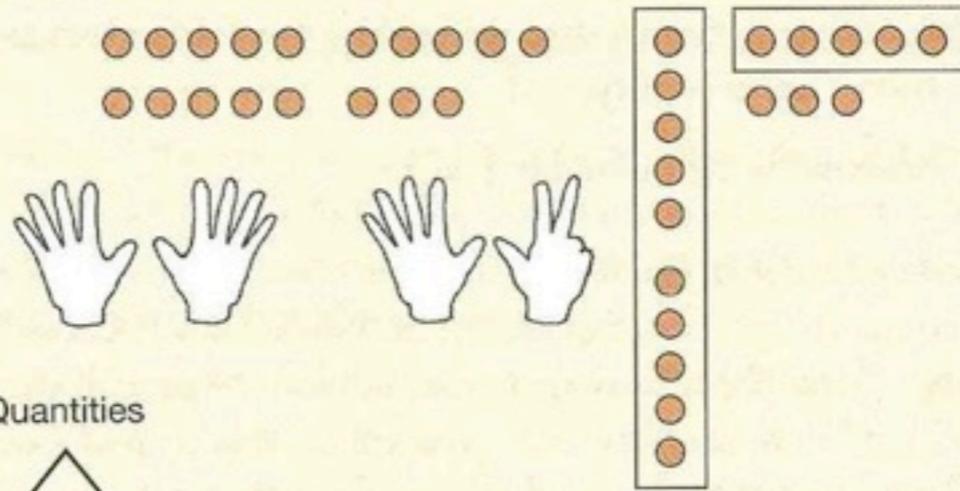
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Quantities

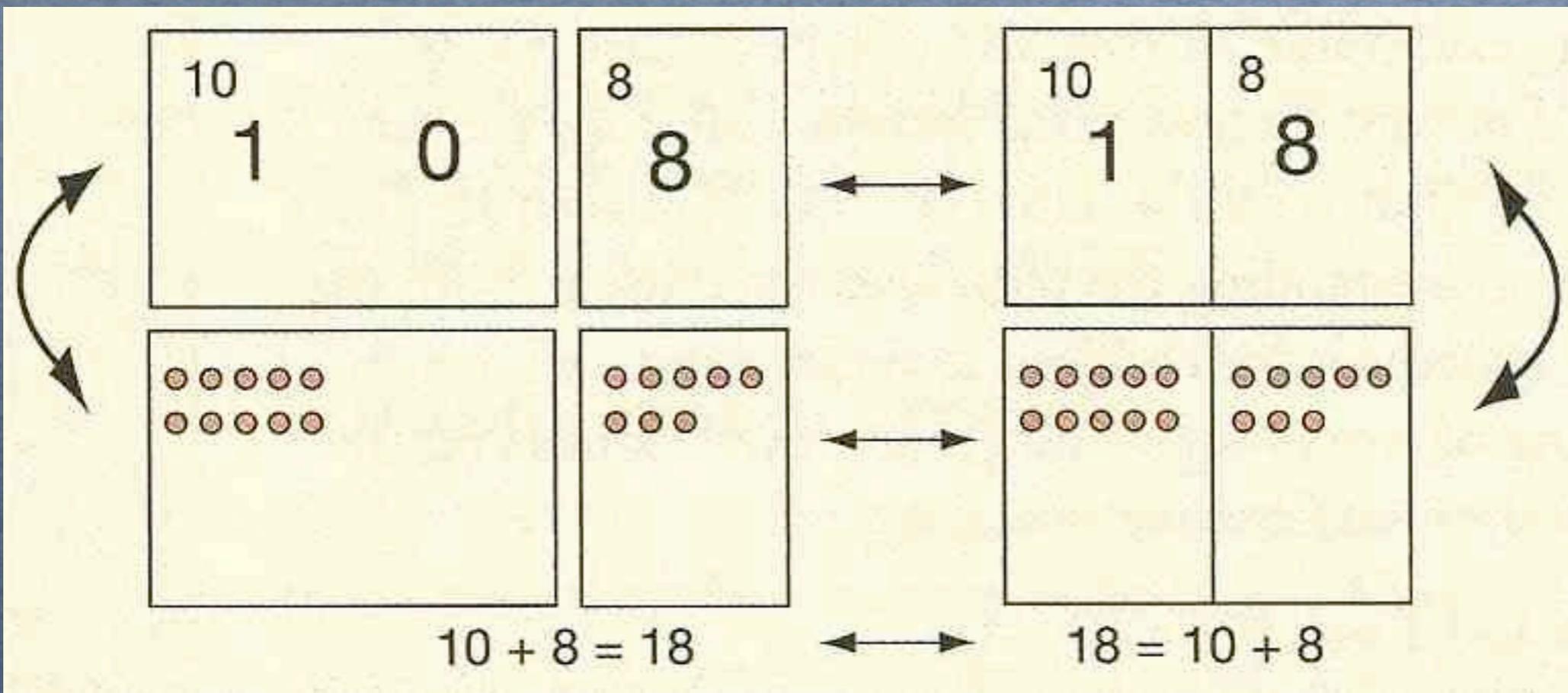
10 to 19

Count Words

Written Numerals

live, thirteen, fourteen, fifteen,  
een, eighteen, nineteen  
en]

The ones are first in the words and  
second in the written numerals.



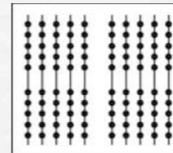
Le carte per i numeri a due cifre

**4 – Comprendere i  
numeri a più cifre e le  
addizioni e sottrazioni  
di numeri a più cifre**

# Fino a 999

## Quantità Parole numerali Simboli scritti

Dare significato ai disegni



Dieci decine e cento unità



Una decina è dieci unità.

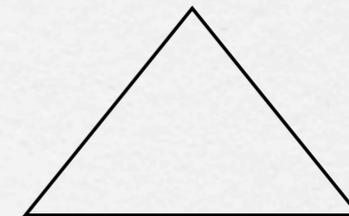
**DISEGNI MATEMATICI (schematici)**



Tre centinaia sette decine nove unità

**PAROLE per indicare QUANTITÀ'**

**Quantità maggiori di dieci**



**Vocaboli numerali**

Valore cui diamo un nome (scrittura lunga)

**Simboli Numerici Scritti**

Cifra singola concatenata

**Schede con il codice segreto**

300
3 0 0

70
7 0

9
9

300	70	9
3	7	9

$$300 + 70 + 9 = 379$$

*Trecento settanta nove*

Impara i cicli ripetitivi nella sequenza del contare nella tua lingua

*Tre sette nove*

Impara a leggere da sinistra verso destra  
La posizione indica il valore

**Numeri maggiori di dieci (visione sequenziale)**

Contare le quantità usando parole numerali combinate (p.e. *duecento trent'uno*) che si accumulano fino a un totale:

Contare per centinaia, poi per decine, e infine per unità, cambiando il conto a ogni nuovo multiplo: 100, 200, 300, (cambiando alle decine) 310, 320, 330, 340, 350, 360, 370, (cambiando alle unità) 371, 372, 373, 374, 375, 376.

**Numeri maggiori di dieci (visione autonoma)**

Conta (vedi) le quantità che comprendono centinaia, decine e unità

**Generalizable and Accessible**  
New Groups Below

Step 1

$$\begin{array}{r} 159 \\ + 187 \\ \hline 346 \end{array}$$

Step 2

$$\begin{array}{r} 159 \\ + 187 \\ \hline 46 \end{array}$$

Step 3

$$\begin{array}{r} 159 \\ + 187 \\ \hline 346 \end{array}$$

**Accessible but Not Generalizable**

Write All Totals  
L → Rt    Rt → L

$$\begin{array}{r} 159 \\ + 187 \\ \hline 200 \\ 130 \\ 16 \\ \hline 346 \end{array} \qquad \begin{array}{r} 159 \\ + 187 \\ \hline 16 \\ 130 \\ 200 \\ \hline 346 \end{array}$$

**Generalizable**  
**Not So Accessible**  
**New Groups Above**

$$\begin{array}{r} 11 \\ 159 \\ + 187 \\ \hline 346 \end{array}$$

**Expanded Notation Form**  
**Either Direction**

$$\begin{array}{r} 159 = 100 + 50 + 9 \\ + 187 = 100 + 80 + 7 \\ \hline 200 + 130 + 16 \\ = 346 \end{array}$$

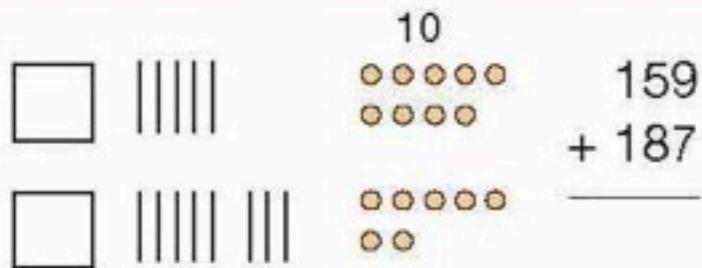
**Not Generalizable**  
**Not So Accessible**  
**Adding On/Recomposing**

$$\begin{array}{r} 159 + 187 = \square \quad (+ 187) \\ 159 + 41 = 200 \quad (+ 146) \\ 200 + 100 = 300 \quad (+ 46) \\ 300 + 46 = 346 \quad (+ 0) \end{array}$$

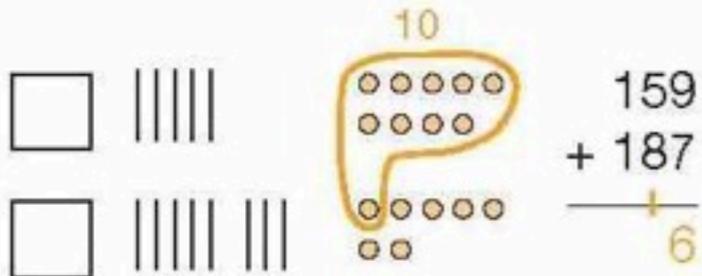
**Scrittura delle addizioni in colonna**

Math Drawing  
and Problem

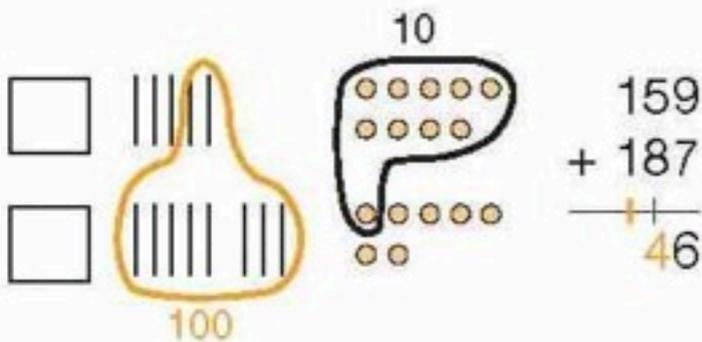
a.



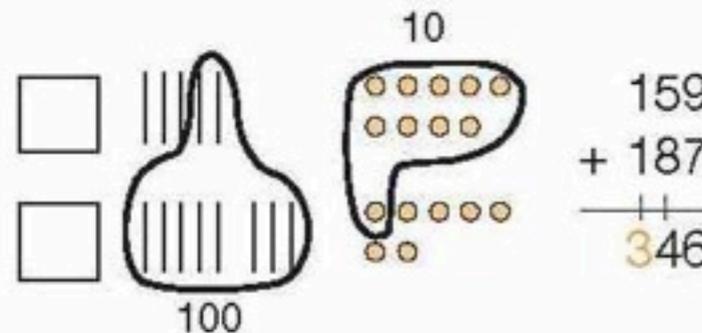
b.



c.



d.



Disegni,  
spiegazioni,  
domande e  
risposte

usando il linguaggio  
posizionale (centinaia,  
decine, unità)

FUSON, Karen 1988 *Children's Counting and Concepts of Number*, New York, Springer

FUSON, Karen 2009 *Avoiding misinterpretations of Piaget and Vygotsky: Mathematical teaching without learning, learning without teaching, or helpful learning-path teaching?*, «Cognitive Development», Vol. 24 (n. 4), pp. 343-361.

FUSON, Karen (con Douglas H. Clements e Sybilla Beckmann) 2010 *Focus in Kindergarten. Teaching with Curriculum Focal Points*, Reston, VA, National Council of Teachers of Mathematics/National Association for the Education of Young Children.